Lecture Note Series 1

Fluid Mechanics

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Contents

Lecture 1 boundary-layer flow

Let us examine a limiting case of the Navier-Stokes equations,

$dw/dt = f - (1/\rho)\ \mathrm{grad}\ p + \nu \Delta w$,

where w is the velocity vector, t is the time, f is the external forces, ρ is the fluid density, p is the pressure, and ν is the kinematic viscosity of the fluid. namely that of very small viscosity or large Reynolds number. We shall explain these simplifications with the aid of an argument which preserves the physical picture of the phenomenon, and it will be recalled that in the bulk of the fluid inertia forces predominate, the influence of viscous forces being vanishingly small.

For the sake of simplicity, we shall consider two-dimensional flow of a fluid with very small viscosity over a horizontal plate, Fig.1.1, about a cylindrical body of slender cross-section, Fig.1.2 and about an airfoil, Fig.1.3, respectively.

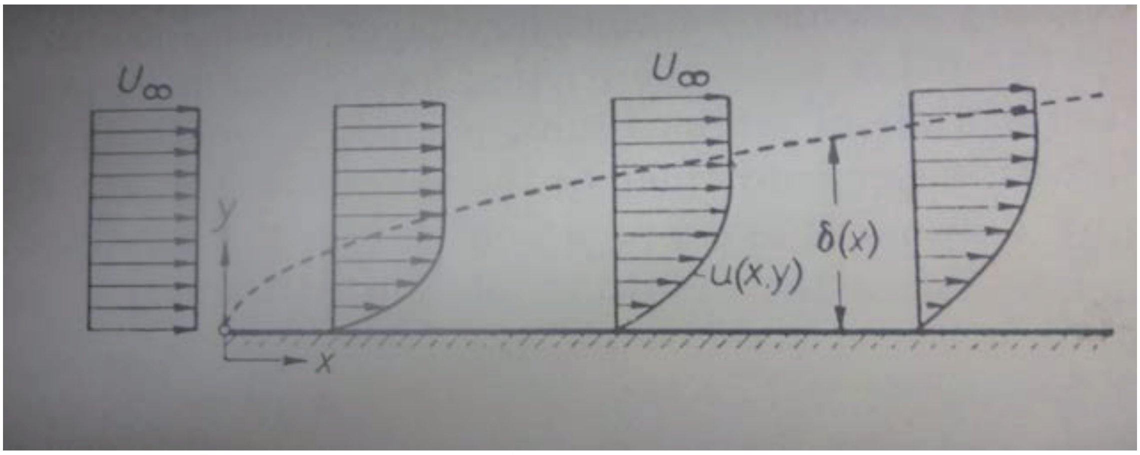

Fig.1.1 Two-dimensional flow with very small viscosity over a horizontal plate

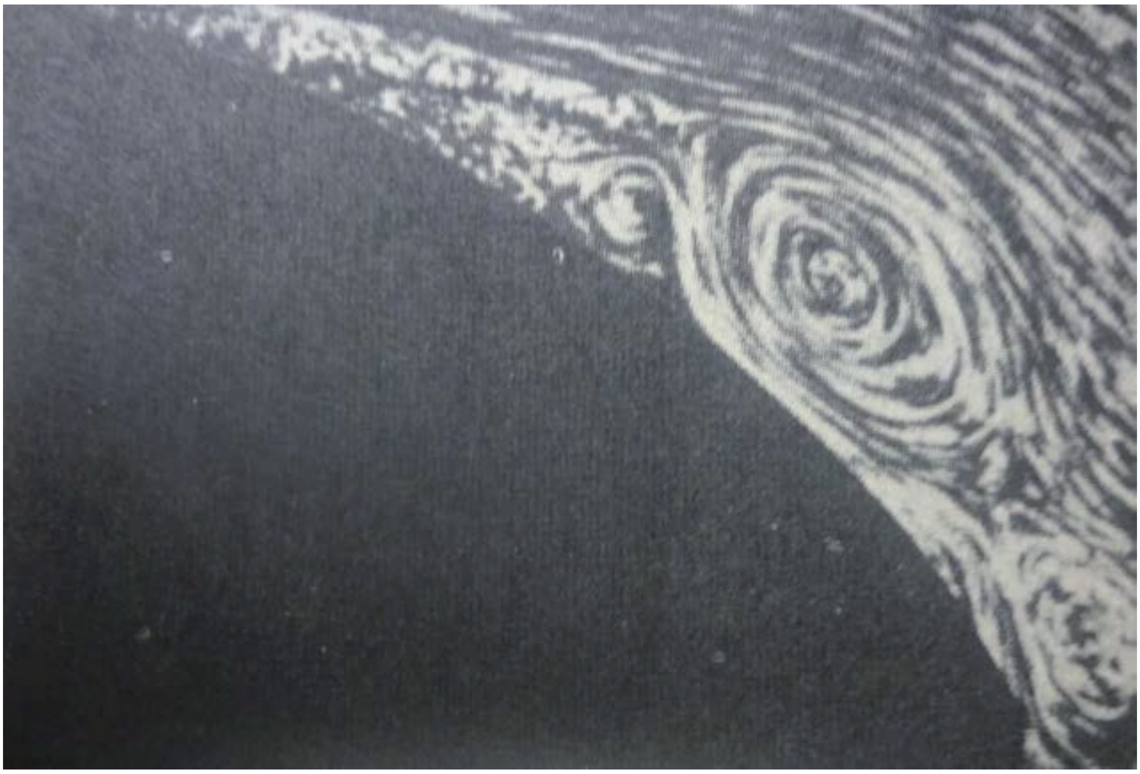

Fig.1.2 Boundary-layer separation over the cylindrical surface.
After Prandtl-Tietjens(1934).

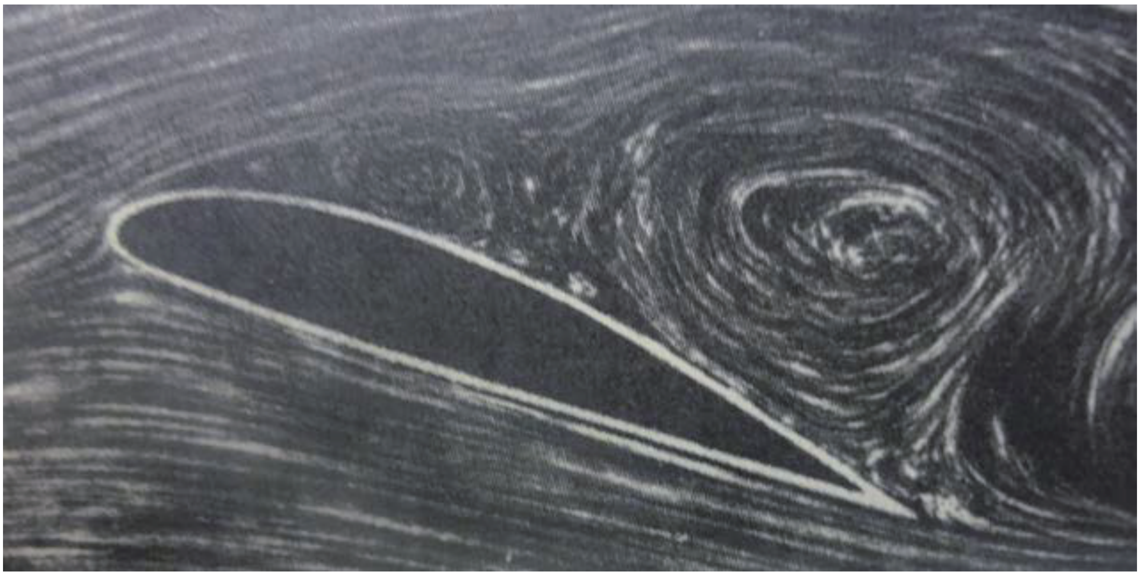

Fig.1.3 Flow around an airfoil when it separates.
After Prandtl-Tietjens(1934)

With the exception of the immediate neighborhood of the surface, the velocities are of the order of the free-stream velocity, V, and the pattern of streamlines and the velocity distribution deviate only slightly from those in frictionless potential flow. However, detailed investigations reveal that, unlike in potential flow to the full magnitude, the fluid does not slide over the wall, but adheres to it. The transition from zero velocity at the wall at some distance from it takes place in a very thin layer, the so-called boundary layer. In this manner, there are two regions to consider, even if the division between them is not very sharp:

1. A very thin layer in the immediate neighborhood of the body in which the velocity gradient normal to the wall, $\partial u/\partial y$, is very large. In this region the very small viscosity μ of the fluid exerts an essential influence in so far as he shearing stress $\tau = \mu(\partial u/\partial y)$ may assume large values.
2. In the remaining region no such large velocity gradients occur and the influence of viscosity is not important. In this region the flow is frictionless and potential.

In general, it is possible to state the thickness of the boundary layer increases with increasing viscosity. It was seen from several exact solutions of the Navier-Stokes equations:

$\delta \sim \nu^{1/2}$.

When making the simplifications to be introduced into the Navier-Stokes equations it is assumed that this thickness is very small compared with a still unspecified linear dimension, L, of the body:

$\delta \ll L$.

In this way the solution obtained from the boundary-layer equations are asymptotic and apply to very large Reynolds numbers.

We shall now proceed to discuss the simplification of the Navier-Stokes equation and in order to achieve it, we shall make an estimate of the order of magnitude of each term. In the two-dimensional problem, we shall begin by assuming the wall to be flat and coinciding with the x-direction, the y-axis being perpendicular to it. Write the Navier-Stokes equations in dimensionless form by referring all velocities to the free-stream velocity, V, and by referring all linear dimensions to a characteristic length, L, of the body, which is so selected as to ensure that the dimensionless derivative, $\partial u/\partial x$, does not exceed unity in the region under consideration. The pressure is made dimensionless with ρV^2, and time is referred to L/V. Further, the expression

$R = VL\rho/\mu = VL/\nu$

denotes the Reynolds number which is assumed very large. Under these assumptions, and retaining the same symbols for the dimensionless quantities as for their dimensional counterparts, we have from the Navier-Stokes equations for plane flow accompanying separation(Fig.1.4):

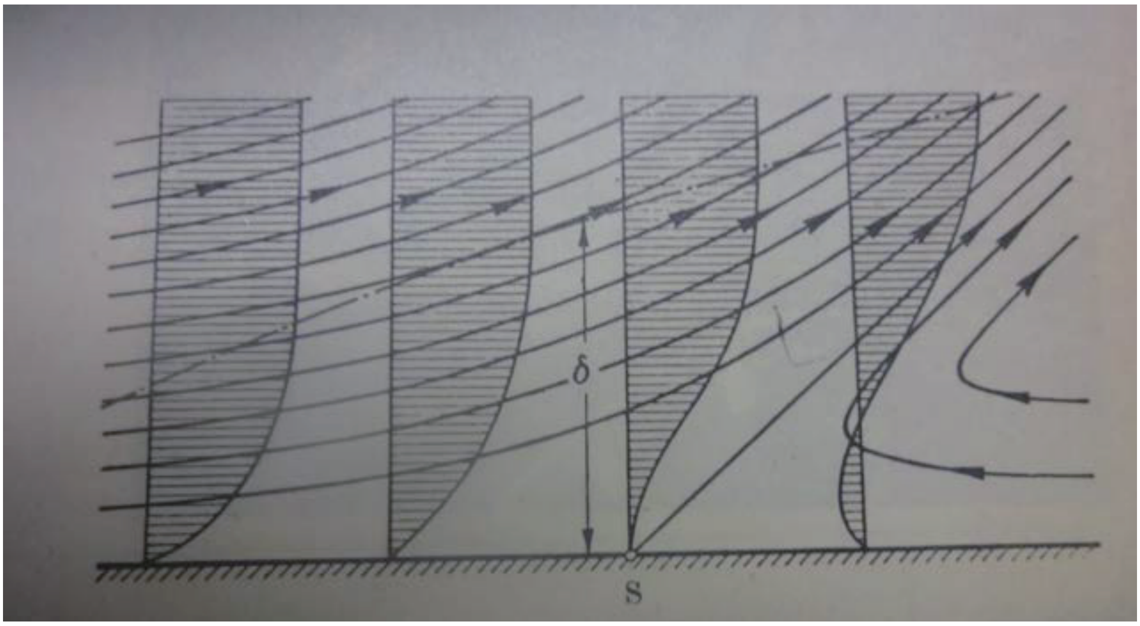

Fig.1.4 Two-dimensional boundary-layer flow accompanying separation in the downstream.

$$\partial u/\partial t + u\,\partial u/\partial x + v\,\partial u/\partial y = -\partial p/\partial x + 1/Re(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2) \qquad (1)$$

$1 \quad 1 \quad 1 \quad \delta \; 1/\delta \quad \delta^2 \quad 1 \quad 1/\delta^2$

$$\partial v/\partial t + u\,\partial v/\partial x + v\,\partial v/\partial y = -\partial p/\partial y + 1/Re(\partial^2 v/\partial x^2 + \partial^2 v/\partial y^2) \qquad (2)$$

$\delta \quad 1 \quad \delta \quad \delta \quad 1 \quad \delta^2 \quad \delta \quad 1/\delta$

continuity equation: $\partial u/\partial x + \partial v/\partial y = 0.$ (3)

$1 \quad 1$

The boundary conditions are: absence of slip between the fluid and the wall, i.e.
$u=v=0$ for $y=0$, and $u=V$ for $y \rightarrow \infty$.

With the assumptions made previously the dimensionless boundary-layer thickness, δ/L, for which we shall retain the symbol δ, is very small with respect to unity, ($\delta \ll 1$).

The orders of magnitude are shown in (1) to (3) under the individual terms. Assume that the non-steady acceleration $\partial u/\partial t$ is of the same order as the convective term $u\cdot\partial u/\partial x$ which means that very sudden accelerations, such as occur in very large pressure waves, are excluded. Some of the viscous terms must be of the same order of magnitude as the inertia terms, at least in the immediate neighborhood of the wall, and in spite of the smallness of the factor 1/R. Hence, some of the second derivatives of velocity must become very large near the wall. This can only apply to $\partial^2 u/\partial y^2$ and $\partial^2 v/\partial y^2$. Since the component of velocity parallel to the wall increases from zero at the wall to the value 1 in the freestream across the layer of thickness δ, we have

$\partial u/\partial y \sim 1/\delta$ and $\partial^2 u/\partial y^2 \sim 1/\delta^2$,

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