Philosophy of Flow, vol.10, Scientific Papers on Turbulence
by Shunichi Tsugé (ed. Takeo R.M. Nakagawa) Supplement,
pp.18 (2013)

The Kolomogorov Turbulence Theory in the Light of Six-dimensional Navier-Stokes' Equation

Shunichi Tsugé

白山連峰学術院出版

Philosophy of Flow, vol.10, Scientific Papers on Turbulence by Shunichi Tsugé (ed. Takeo R.M. Nakagawa) Supplement , pp.18 (2013)

Online Journal : arXiv:nlin.CD/0303013 v2 12Mar2003

The Kolmogorov Turbulence Theory in the Light of Six- dimensional Navier - Stokes' Equation

Shunichi Tsugé
790-3 Tohigashi, Tsukuba 300-26 Japan
tsuge@nyc.odn.ne.jp

Abstract: The classical turbulence theory by Kolmogorov is reconsidered using Navier-Stkes' equation generalized to 6D physical-eddy space. Strong pseudo-singularity is shown to reveal itself along the boundary 'ridge' line separating the dissipation and inertial sub-ranges surrounding the origin of the eddy space. A speculation is made that this singularity is generated by two dipoles of opposite sign aligned on the common axis. It is supported by the observation that the universal power spectrum calculated rediscovers the Kolmogorov's -5/3 power law as independent of the dimensional approach.

1. Introduction

As early as in 1941 Kolmogorov [1] predicted some universal features of fluid turbulence which have been confirmed by experiments in later years. It is rather surprising that they are derived from dimensional analysis based on the simple assumption of 'local homogeneity' for small-scale turbulence. More surprising is the fact that those have little to do with the equation of dynamics of fluid.

It is addressed in this paper to locate this 'missing link' through re-derivation of the -5/3 power law of the spectrum for the inertial sub-range as the universal law using the dynamical equation proposed in ref.2. The key issue for this task to work is that equation on which to describe the features of small eddies as independent of individual flow geometry. To meet this purpose a six-dimensional Navier-Stokes equation is employed, where additional 3D space having a length dimension, corresponding to eddy size, is introduced.

Originally such an equation has been derived elsewhere [2], using non-equilibrium statistical mechanics starting as natural consequence of a mathematical procedure, namely, the separation of variables of turbulent fluctuation-correlation equation. The equation stands as 6D generalization of the Navier-Stokes equation which in 3D physical space degenerates to the classical equation.

In what follows, however, it is intended to re-derive the same equation using phenomenologies alone on the basis same as the classical theory of Kármán and Howarth [3]. Then, the wave-number space is introduced for separating variables (sec.2). It is then Fourier-transformed into eddy space, thereby the 6D Navier-Stokes equation, the basis to all what follows is established (Sec.3). The equation gives a novel expression for turbulent dissipation, enabling to predict existence of a pseudo-singularity surrounding the dissipation region (Sec.4). In inertial sub-range, this singularity turns out to be a couple of dipoles of opposite sense, separated by the order of several tens to hundreds the Kolmogorov length(Sec.5). Power spectrum of this locally homogeneous turbulence is calculated, showing a wav-number dependence close to -5/3 power law of the Kolmogorov theory (Sec.6).

2. Kármán-Howarth formalism revisited as governing inhomogeneous turbulence

In 1938, von Kármán and Howarth proposed an equation governing homogeneous isotropic turbulence whose original form is written as[3]

$$\langle u_j'(\hat{NS})_\ell + \hat{u}_\ell' (NS)_j \rangle = 0, \tag{1}$$

In this equation $u_j' = u_j'(\mathbf{x}, t)$ and $\hat{u}_\ell' = u_\ell'(\hat{\mathbf{x}}, t)$ are instantaneous velocity fluctuation at $\mathbf{x}$ and $\hat{\mathbf{x}}$, respectively, the symbol $\langle\rangle$denotes the ensemble average, and

$$\mathbf{NS} \equiv \mathbf{NS}(\nabla, \underline{\mathbf{u}}, \underline{p})$$

$$= (\partial / \partial t + \underline{u} \cdot \nabla - \nu \nabla^2)\ \underline{\mathbf{u}} + 1/\rho \cdot \nabla \underline{p} = 0 \tag{2}$$

denotes the Navier-Stokes equation written in terms of the instantaneous fluid quantities

$$\underline{\mathbf{u}} = \mathbf{u} + u',$$

$$\underline{p} = p + p', \tag{3}$$

where $\mathbf{u}$ and p are the average velocity and pressure, respectively, ν is the kinematic

続きは

完成版で

お楽しみ下さい。